NOTES

Fibonacci/Lucas Modulo Cycles

Six centuries apart, Leonardo of Pisa "Fibonacci" and Edouard Lucas discovered a pair (one each) of additive integer series whose generative rules might be combined as follows:

> Each next term sums the current term and its immediate predecessor. Term #1 is always 1. Term #0 is either one less (0, for the Fibonacci) or one more (2, for the Lucas).

In the illustration below, Term #0 is at the center, and addition proceeds from there to the right. A reverse process, to the left instead by subtraction, reveals the mirror at this center term.

ID#:	...	−6	−5	−4	−3	−2	−1	0	1	2	3	4	5	6	...
								|							
Fib:	...	−8	5	−3	2	−1	1	0	1	1	2	3	5	8	...
Luc:	...	18	−11	7	−4	3	−1	2	1	3	4	7	11	18	...

As defined, the series continue infinitely. Intending to harness these as musical source material, however, I needed first to render them in finite form. Modular (remainder only) division offered an organic means to achieve this. Expressed as a list of remainders from each term's division by, for example, a modulus of 4, the above segments became:

Fib:	...	0	1	1	2	3	1	0	1	1	2	3	1	0 ...
Luc:	...	2	1	3	0	3	3	2	1	3	0	3	3	2 ...

Thus infinite series are converted to repeating (finite) cycles. There is no limit on cycle length (here 6), and a given cycle's content is unique to the modulus invoked.

Noting in such cycles the frequency of individual members' occurrence and comparing results to the cycles themselves, one sees three types of correspondence. Against its source cycle, each occurrence frequency pattern either:

A) has full–cycle length and iterates once;

Fib Mod 11: 1 1 2 3 5 8 2 10 1 0
Occur Frq: 3 3 2 1 1 1 2 1 3 1

B) has half–cycle length and iterates twice;

Fib Mod 8: 1 1 2 3 5 0 , 5 5 2 7 1 0
Occur Frq: 3 3 2 1 3 2 , 3 3 2 1 3 2

C) or has quarter–cycle length and iterates 4 times.

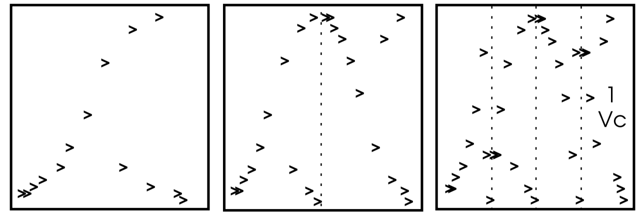
Fib Mod 13: 1 1 2 3 5 8 0 , 8 8 3 11 1 12 0 , ...
Occur Frq: 4 4 2 2 4 4 4 , 4 4 2 2 4 4 4 , ...

12 12 11 10 8 5 0 , 5 5 10 2 12 1 0
4 4 2 2 4 4 4 , 4 4 2 2 4 4 4

These subdivision types are illustrated graphically on the next page. Within each graph: mod–cycle values, seen as angle brackets, range along the vertical axis; occurrence frequencies, seen as distances between consecutive angle–marked columns, are on the horizontal. The distances are mapped as reciprocals so that for each cycle value, however often or seldom occurring, total accumulated distance is the same.

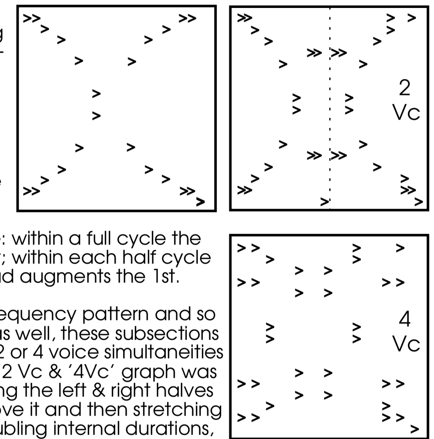
I have adopted Fib/Luc mod cycles, posited so, as the basis for Additudes!' melodic material, mapping cycle terms and their occurrence frequencies as proportions respectively for pitch and rhythm. These next acquire absolute audio specs through criteria developed on a separate, formal basis: the Order–21 Perfect Square.

A (Mod 29) B (Mod 18) C (Mod 17)



Every Fibonacci cycle section corresponding to one iteration of occurrence frequencies ends with '0', as seen here in each graph's lowest level 'pluck'.

Demarcated by such zero points (and some dotted lines) the subsections at page top are seen to interrelate: within a full cycle the 2nd half inverts the 1st; within each half cycle the 2nd quarter instead augments the 1st.



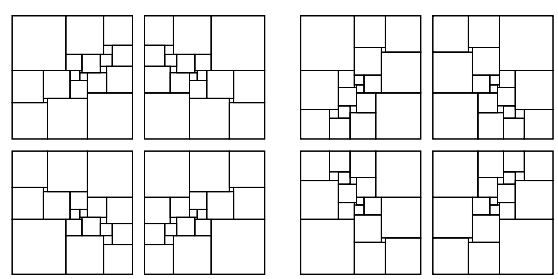
Sharing occurrence frequency pattern and so rhythmic proportions as well, these subsections combine to produce 2 or 4 voice simultaneities when overlaid. Each '2 Vc & '4Vc' graph was made by superimposing the left & right halves of the pattern just above it and then stretching the result laterally, doubling internal durations, to refill the original time dimension.

An additional category of overlays is offered by the 'N' densities, only occasionally "accidentally" available. Their chords, synchronizing up to a hundred events, are heard as percussive timbres.

The Order–21 Perfect Square

A square is 'perfect' if tiled entirely by unique–sized squares. While only a few instances of such a figure have been discovered, it is known that its tiles must number no fewer than 21, and that 21 permit just one selection of sizes in exactly one configuration.

The CD case front panel displays this figure, simplest perfect square. Its area sums the tile areas; its side sums the tile sides through any horizontal or vertical cross section. The framing side measures 112 units; the tile sides 2 4 6 7 8 9 11 15 16 17 18 19 24 25 27 29 33 35 37 42 50. The tile configuration while singular is also asymmetric, yielding fully 8 unique structural variants through re–orientation alone: 4 mirror images, and the same with their axes swapped.



I have taken the figure to underlie Additudes!' outer structure, mapping its frame and tile dimensions to sound parameter *ranges.* The frame maps vertically to a pitch span of 6 octaves, horizontally to a time span of 60 seconds. In this context each tile square becomes the absolute pitch/time setting for one of the proportional Fibonacci/Lucas array pairs derived earlier. Indeed, these 21 tile sides provided all moduli for the original cycle generations.

Explanation of Names

0 0 0 0 ... 1 1 1 1

These binary number look–alikes announce the voice densities to be heard in each movement. Their digits are positional flags standing right to left for 1, 2, 4 & N (other). Each name occurs once only, representing a unique *combination* of densities. In a movement for each of the 16 possible combinations, cycle settings are heard against pulsation tracing the sides of the tile squares themselves.

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As the graphic suggests, these sections function as separators. Each sets off a subset of the digit–named movements, which are grouped by the *number of* density types they combine. Melodic contours here are flattened to near monotones and their rhythms disrupted – mapped now from the mod cycles directly – to evoke fragmented straight lines.

E p i l

From whirl to whimper the Epilogue acts as contrarily as it can. Ignoring Additudes!' cohesive principle – the framing square – it attacks the tile areas one by one, saturating each with an overlay of every voice density possible. The result yields two opposing rhythmic surprises: regularity, wherever densities fall within 1,2,4; chaos, where they are "N" instead.

Additudes!

On its face this caricatures folks with attitudes saying "attitudes". More substantially, it names an essence that binds this music's concepts: the idea of adding.

The Csound Orchestra

This work was synthesized via Csound, a digital–audio program suite created in the '70s by Barry Vercoe of MIT and presently maintained by John ffitch at University of Bath with continuing development by Dr. Richard Boulanger, Michael Gogins, and many others.

Csound reads two files: an orchestra (*.orc) defining software instruments, and a score (*.sco) specifying when and what the instruments are to "play". Additudes!' orchestra consists of a single instrument built around 'pluck' – Csound's digital model for the plucked string – and enhanced with a string resonator.

Expanding here briefly for Csounders: pluck is invoked with its decay method always '3' ("simple drum") but with a roughness factor never greater than 0.04 – that is, short of masking pitch. Adjusting within this limit for duration, range and voice density, the factor subsequently sets streson's fdbgain as well.

The 'J' Programming Language

Additudes!' 21 scores were composed algorithmically. Each is my pick from several output under its movement's constraints and alternatives by "additudes.ijs", a J script of some 800 lines. The J language, by Kenneth Iverson & Roger Hui, is a superset of and successor to Iverson's APL, which first treated the multi–dimensional array as a primitive data type. The script implemented all logic I have described, formatted the event specs ('i' commands of 27 elements), and wrote the .sco files for Csound.

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